

# Physical model of piano tuning

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## Abstract

We construct a model of tuning a single piano string taking into account the friction points of the string as well as the twisting of the pin. The pin angle is controlled by a tuning lever which is moved to change the angle of the pin. This pulls the string which then moves and it also introduces a torsoidal deformation of the pin (“twist”). So if the lever is released the twist will unwind and potentially the pitch can shift. In addition the application of a “test blow” (playing the note very loud) may cause the string to shift at the friction point, causing another shift in pitch. The goal is to arrive at a desired pitch which is stable under test blows. Numerical simulation of the model is performed from which we derive stable tuning techniques which depend on the details of the friction and torsion parameters of the piano.

## 1 Introduction

With the popularization of the piano around the turn of the previous century a new profession appeared: the professional piano tuner. Stringed instruments have been used for millennia and have always required frequent tuning, which was one of the tasks of the player. The large tensions and large number of strings of the piano made this task sufficiently difficult that nowadays pianos are tuned by professionals.

Deciding on the pitch of each note of the piano is non-trivial due to inharmonicity which causes the harmonics to deviate from the natural overtone series [3]. Many schemes for tuning are used and often passionately defended, but in this article we will not be concerned with this issue and consider just the task of tuning a specific piano string to a specific pitch.

This task is accomplished by turning the pin with a tuning lever (usually called “hammer”). The pin is a cylinder which is held in place in a piece of wood called the pin block. The string is wound around the pin and by turning it the string length can be changed. The sounding part of the string is not directly attached to the pin but passes over several pressure points where friction is generated. These friction points as well as the rotational deformation of the pin due to the torque exerted by

the tuner cause the relation between pitch and lever to be rather complicated. In particular, due to friction, the string can potentially shift after tuning if tensions are not properly equalized. The difficulty is to arrive at the correct pitch in a stable manner, meaning the pitch will stay there during playing. A plethora of “hammer techniques” are in use by professional piano tuners, each advocating superiority in stability. See for example [1].

In this article we will create a model of the tuning process taking into account the most important factors governing the process. It is shown that details specific to a particular piano require different strategies to arrive at a stable tuning.

## 2 The pin model

The tuning pin is modeled as a cylinder of radius  $r_p$ . One side of the pin resides in the pin block, and the string is attached somewhere on the part that sticks out. Clearly if the pin is rotated by an angle  $\theta$  (in radians), the string is shortened or lengthened (depending on how the string is wound) by a factor  $r_p\theta$ . Due to the non zero thickness of the string  $r_p$  should be taken to be the radius of the pin itself plus half the radius of the string. In this paper we shall assume an upright piano configuration where the string is wound on the pin so that when the pin is turned counter clockwise (the positive direction in our convention) the string unwinds and the pitch goes down.

Twisting the pin is a complicated process as the segment inside the pin block can have a complicated friction torque distribution. We simplify this by assuming the pin is effectively held in place at a distance  $L_p$  from the string attachment and we compute the twisting based on this model.

When the pin is twisted a torque is exerted on the pin block and if this exceeds a threshold, the pin turns. We shall assume a Coulomb model for the frictional process which means that the maximum torque  $\tau_B$  exerted on the block is limited to

$$|\tau_B| < \tau_{max}. \quad (1)$$

The parameter  $\tau_{max}$  is not very intuitive and we shall parametrize it in terms of the “pin tightness” coefficient  $\sigma_p$ :

$$|\tau_{max}| = \sigma_p r_p F_0 \quad (2)$$

where  $F_0$  is the “usual” string tension (in Newtons). During tuning it changes only a little bit from  $F_0$  so we consider it constant. Clearly we must have  $\sigma_p > 1$  or the pin would turn by itself.

The state of the pin is described by two angles  $\theta_H$  (the angle at the string attachment) and  $\theta_B$  (the angle at the effective base). We shall denote the twist angle by  $\psi = \theta_H - \theta_B$ . The torque  $\tau_B$  is now given by

$$|\tau_B| = \frac{J\mu}{L_p}\psi \quad (3)$$

with  $\mu$  the shear modulus (force/area) of the pin material and  $J = \pi r_p^4/2$ . The shear modulus of steel is  $\mu = 80GN/m^2$ , but our approximation of the pin block contact and the fact that additional twist could be caused by wood deformation makes this an uncertain parameter. Because the shear modulus enters only in the form of the ratio  $\mu/L_p$  we adopt the convention that  $\mu = 80GN/m^2$  is fixed whereas we consider  $L_p$  to be an uncertain parameter, to be fitted to measured data.

### 3 The string model

The string is modeled as a cylinder of radius  $r_s$ , mass density  $\rho$ , and tensile forces are determined by Young's modulus  $E$ , which is about  $150 - 210GN/m^2$  for spring steel. As indicated in Fig. 1 the string is divided into speaking and a non-speaking parts with lengths  $L_n$  and  $L_s$ . The string passes over the V-bar, which deflects the string from a straight line from bridge to pin by amount  $d_V$ . There is an additional contact point, the pressure bar, indicated in the figure, which we ignore in our model as the friction force is much less at that point than at the V-bar. The change in tension  $\Delta F$

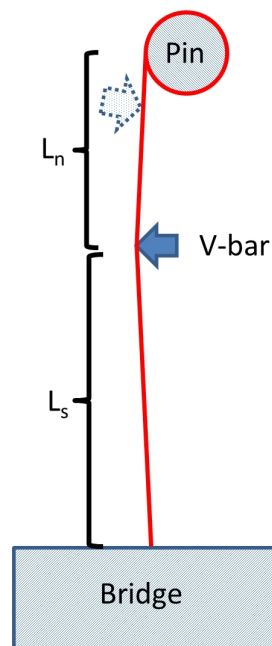


Figure 1: String is attached to the bridge, runs over the V-bar which is raised a distance  $d_V$  and attached to the pin. The upper segment of the string is the non-speaking segment of length  $L_n$  and the lower segment of length  $L_s$  is struck by the hammer. The tensions in the two segments are  $F_n$  and  $F_s$ . The pressure bar that we neglect in the model is indicated by the dotted arrow.

in a string segment of length  $L$  when stretched by a small amount  $\Delta L$  is given by

$$\Delta F = \kappa \Delta L / L. \quad (4)$$

with

$$\kappa = \pi r_s^2 E \quad (5)$$

the spring constant of the string. For a string of length  $L$  under tension  $F$  the relation between sounding frequency and tension is given by Mersenne's law

$$F = \pi r_s^2 \rho (2Lf)^2. \quad (6)$$

If the tension changes by an amount  $\Delta F$  the sounding pitch changes by an amount  $\Delta f$  determined by the simple equation

$$\frac{\Delta f}{f} = \frac{\Delta F}{2F}. \quad (7)$$

In terms of cents this reads

$$c = \frac{600}{\log(2)} \frac{\Delta F}{F}, \text{ where} \quad (8)$$

$$c = \frac{1200}{\log(2)} \log((f + \Delta f)/f). \quad (9)$$

The friction force at the V-bar is modeled as Coulomb friction and limited to a maximum value  $F_V$ , i.e.,

$$|F_n - F_s| < F_V. \quad (10)$$

If the coefficient of friction is  $\gamma$ , we can compute  $F_V$  if we know the normal force and obtain

$$F_V = \gamma \frac{d_V}{L_s} F_0 \quad (11)$$

which is valid for small  $d_V$ . Note that this depends only on the product  $\gamma d_V$  so we define a reasonable value for  $d_V = 1\text{cm}$  which we consider fixed, and consider  $\gamma$  to be the uncertain parameter, to be fitted by measurements.

During loud playing (or by applying a test blow) the vibrations in the string reduce the coefficient of friction, causing a possible shift in pitch. We describe this process by a factor  $0 \leq s_f < 1$  which multiplies the coefficient of friction during playing.

An additional factor that comes into play during a test blow is the increase in tension in the string when excited loudly. This introduces a non-linear effect which has a complicated effect on the vibration [2]. For our purposes all we need to know is how much the effective string tension in the speaking segment increases at a test

blow as this will apply an additional force across the V-bar. A consequence of this phenomena is the well-known effect that a test blow usually lowers the string pitch when the tuning is not stable, but never or rarely raises it. Without incorporating this non-linear effect the pitch would raise on a test blow if the non-speaking segment had a higher tension, which is not observed. We describe this effect by a dimensionless parameter  $\beta$  which increases  $F_s$  to  $(1 + \beta)F_s$  during a simulated test blow.

## 4 Measuring the parameters

The model we describe in the previous sections contains 13 numerical parameters, and here we describe how to obtain them. Starting with the easiest ones, the V-bar elevation  $d_V$  and the pin shear modulus can be fixed to  $d_V = 1cm$  and  $\mu = 80GN/m^2$  as these only appear in the model in the form of the product  $d_V\gamma$  and the ratio  $\mu/L_p$ . Next we measure the string speaking and non-speaking lengths with a tape measure, and the pin and string diameters with a micrometer. The pin diameter is best measured around the string coil and the model pin radius  $r_p$  is then just half the measured diameter minus half the string diameter, i.e.,  $r_p$  is the radius of the coiled string center.

The mass density  $\rho$  of the string has the value for spring steel,  $\rho = 7850kg/m^3$ . This value varies only a little for various types of steel. To obtain the pin stiffness  $\sigma_p$  we need to take torque measurements, which will also give us a check on  $\rho$ .

With a torque wrench we measure the torque  $\tau_L$  required to turn the pin counter clockwise, and the torque  $\tau_R$  required to turn clockwise. We'll have  $\tau_L < \tau_R$  as the string applies its own torque to the pin. Because of the geometry of the torque wrench used, some lateral force is applied to the pin as well, which could affect the readings, so we can not expect too much accuracy. We have

$$\tau_L = \tau_{max} - r_p F_0 \quad (12a)$$

$$\tau_R = \tau_{max} + r_p F_0 \quad (12b)$$

from which we obtain

$$r_p F_0 = \frac{1}{2}(\tau_R - \tau_L) \quad (12c)$$

$$\sigma_p = \frac{\tau_R + \tau_L}{\tau_R - \tau_L}. \quad (12d)$$

Measurements on the three A4 (440Hz) strings of a Heintzmann upright piano gave  $r_p = 3.9mm$ ,  $r_s = 0.5mm$ ,  $L_s = 37.5cm$  and the torques for the left, middle and right strings were  $\tau_R = (13.6 \ 14.1 \ 15.3)Nm$  and  $\tau_L = (8.1 \ 9.0 \ 10.2)Nm$ . The tensions derived from this are  $F_s = (705 \ 654 \ 654)N$ . Using Mersenne's law (6) this gives us three estimates of the mass density,  $\rho_{estimated} = (8244 \ 7645 \ 7645)kg/m^3$ , which are close enough for comfort to the theoretical value of  $\rho = 7850kg/m^3$ . The values of the pin stiffness coefficient are obtained from (12d):  $\sigma_p = (3.9 \ 4.5 \ 5.0)$ .

The value of Young's modulus is obtained by recording the angle  $\phi$  by which we have to turn the pin to lower the pitch by a semitone. Using (4), (5), and (8), and  $\Delta L = r_p \phi$  we can solve for  $E$ . For the three A4 strings we measured  $\phi = (3.6 \ 2.9 \ 2.9)^\circ$  from which we obtain  $E = (159 \ 183 \ 183)GN/m^2$ , which are reasonable values. We shall use the average of the three,  $E = 175GN/m^2$ .

The four remaining parameters are the string coefficient of friction  $\gamma$ , the effective pin length  $L_p$ , the factor  $\beta$  which parametrizes the non linear tension increase during a test blow, and the friction reduction during test blow factor  $s_f$ . We find values for these parameters by running the simulation and adjusting them by trial and error to match observed behavior under the following tests.

We lower (raise) the pitch a bit using a smooth motion, then hold the tuning lever fixed (i.e., under torque) and apply a test blow. The shift in pitch after the test blow is then measured. The same experiment is performed but instead of holding the lever fixed we release it. For the piano under consideration no pitch shift was observed when raising pitch under both conditions, whereas going the opposite direction the pitch dropped by 3¢ when fixing the lever and by 2¢ otherwise. The reason for the difference is the untwisting of the pin when we release the lever which increases the tension (which was lower than in the speaking section) in the non-speaking segment, hence leading to a smaller drop in pitch. The fact that the pitch was not observed to raise when holding the lever fixed when raising pitch, even though the tension in the non-speaking section is as high as it can be can only be explained by the effect of the parameter  $\beta$ . In the case of releasing the lever first the untwisting of the lever will also counteract the tendency for the pitch to rise as it will reduce  $F_n$ .

We list the values used for the left A4 string in Table 1.

## 5 Tuning up (down)

Suppose everything is in equilibrium, we have  $F_n = F_s$ , and the pin will have a certain (positive) twist  $\psi$ .

Assume the pitch is too low (high) and we raise (lower) it by slowly turning the pin (counter) clockwise. Note that we position the pin with the lever, so the torque on the pin block is uniquely determined and not dependent on the string. At first the tension  $F_n$  in the non speaking segment will rise (drop), but it will be balanced by a downward (upward) friction force at the V-bar. So  $F_s$  and the pitch does not change. Once the tension differential across the V-bar becomes larger than  $F_V$ , the maximum friction force, the string will slide up (down) over the V-bar, thereby increasing (decreasing) the tension  $F_s$  and raising (lowering) the pitch. Since now  $F_n = F_s + F_V$  ( $F_n = F_s - F_V$ ), the maximum differential force allowed, a test blow at this point (without moving the lever) would result in a reduction in tension differential, so  $F_s$  would increase (decrease) and the pitch would go up (down). The reason was the higher (lower) tension in the non-speaking segment.

	A4
$f$	440Hz
$L_s$	37.5cm
$L_n$	10cm
$r_s$	0.5mm
$(d_V)$	1cm
$E$	175GN/m <sup>2</sup>
$\rho$	7850kg/m <sup>3</sup>
$r_p$	3.9mm
$\sigma_p$	4.1
$(\mu)$	80GN/m <sup>2</sup>
$L_p$	2mm
$\gamma$	0.08
$\beta$	0.008
$s_f$	0.4

Table 1: Parameters values. Red variables are pin parameters. Values below the horizontal line are uncertain, in that they can not be measured directly. As discussed before the V-bar elevation  $d_V$  and the pin shear modulus  $\mu$  can be considered given (hence parenthesized in the table) as they only enter in the model through the product  $d_V\gamma$  and the ratio  $\mu/L_p$ .

If instead we remove the lever something else happens. Because we have turned the front of the pin (counter) clockwise the base lags a bit behind, in other words we will have a negative (positive) twist  $\psi$  which is at its maximum value assuming the lever was turned enough to cause the pin base to move. If we now let go of the lever, the string will pull the pin counter clockwise (clockwise) to bring the twist back to its equilibrium value. This will reduce (increase)  $F_n$  by a certain amount. This can be a positive thing, as when we now apply a test blow the tension differential may be small enough to keep the note at pitch. However it may not reduce (increase)  $F_n$  enough in which case the pitch will raise (drop). Another possibility is that this reduces (increases)  $F_n$  so much to change the sign of the tension differential so that  $F_n < F_s$  ( $F_n > F_s$ ) after removing the lever. This may cause an immediate drop (raise) in pitch or a drop (raise) after a test blow.

Note that when we tune up the effect of pitch twist is much stronger as the existing positive twist due to the string tension has to be reversed whereas when tuning down the twist is in the same direction.

## 6 Tuning simulation

We model the tuning process by prescribing a lever trajectory  $\theta(t)$  over “times”  $t$  and computing what happens to the system. Let us assume we are in stable configuration described by the state variable vector  $S = (F_n, F_s, \psi)$  and compute what happens when the pin is rotated by a very small amount  $\Delta\theta$ . We then divide the lever trajectory in small segments and compute the state over the discretized trajectory on step at a time. We will also want to calculate what happens at each point in the trajectory under three actions: 1) release the lever, 2) release the lever and apply a test blow, and 3) apply a test blow while holding the lever in place. The third action never occurs in normal tuning, but we will compute the result here anyways as this action is useful to calibrate the uncertain parameters of the model.

### 6.1 Turning the lever

We start by computing what happens if we just turn the lever. When the lever turns by  $\Delta\theta$  the twist changes according to

$$\hat{\psi} = \psi + \Delta\theta. \quad (13)$$

(Hatted quantities indicate the new value.) The maximum allowed twist magnitude is

$$\psi_{max} = \frac{L_p}{J\mu} \tau_{max} \quad (14)$$

according to (2) and (3). So if  $\hat{\psi} > \psi_{max}$  we set  $\hat{\psi} = \psi_{max}$ , if  $\hat{\psi} < -\psi_{max}$  we set  $\hat{\psi} = -\psi_{max}$  (physically the base of the pin turns), and otherwise it becomes (13). Next we compute the new force  $\hat{F}_n$  assuming the string does not shift at the V-bar. If it turns out that

$$|\hat{F}_n - F_s| \leq F_V \quad (15)$$

with  $F_V$  given in (11) we are done. The change in length of the non-speaking segment is

$$\Delta L = r_p \Delta\theta \quad (16)$$

so according to (4) the change in  $F_n$  is

$$\Delta F_n = -\kappa \Delta L / L_n \quad (17)$$

and  $F_s$  does not change.

If (15) is not satisfied we have to calculate the shift  $\Delta V$  over the V-bar which will be positive (up) if  $\hat{F}_n - F_s > F_V$  and negative (down) if  $F_s - \hat{F}_n > F_V$ . The former



can happen when the pin turns clockwise,  $\Delta\theta < 0$ . In that case the length change will be  $\Delta L + \Delta V$  and the new force is

$$\hat{F}_n = F_n - \kappa(\Delta L + \Delta V)/L_n. \quad (18)$$

The new force in the speaking segment is now

$$\hat{F}_s = F_s + \kappa\Delta V/L_s. \quad (19)$$

Finally the tension differential must be maximal (i.e., the shift stops when the static friction stops the process) which gives the condition

$$\hat{F}_n - \hat{F}_s = F_V \quad (20)$$

from which we can solve for  $\Delta V$  giving

$$\Delta V = \frac{L_{ns}}{\kappa}(F_n - F_s - F_V) - \frac{L_{ns}}{L_n}\Delta L \quad (21)$$

with  $\Delta L$  given by (16) and  $L_{ns}$  is the harmonic average of the string lengths

$$L_{ns} = 1/\left(\frac{1}{L_s} + \frac{1}{L_n}\right). \quad (22)$$

New updated forces are now obtained from (18) and (19). Repeating this for the down case ( $F_s - \hat{F}_n > F_V$ ) gives the same formulae except in (21) we have to change the sign in front of  $F_V$ .

## 6.2 Releasing the lever with or without test blow

Next we determine how to compute the state change when the lever is removed perhaps followed by a test blow. This will determine the final pitch. When the lever is removed the pin will twist back to its natural position. When that happens the length of the non-speaking segment changes, hence the force and this may or may not cause the string to slide over the V-bar, thereby changing the final pitch. If we consider a simultaneous test blow we just have to multiply  $F_V$  by a factor  $s_f$  and recall that the downward pull across the V-bar from the speaking segment is now  $(1 + \beta)F_s$ . In the analysis below we will assume a test blow. To obtain the effect of releasing the lever without a test blow just set  $\beta = 0$  and  $s_f = 1$  everywhere.

As before we first pretend that there is no shift at the V-bar and calculate the new force  $\hat{F}_n$ . If it turns out that

$$|\hat{F}_n - (1 + \beta)F_s| \leq s_f F_V \quad (23)$$

holds we are done, if not we have to incorporate a shift  $\Delta V$  such that all forces balance and the tension differential is maximal. For the first case (i.e., (23) holds)

the pin base torque and torque caused by the string have to be equal. If the pin turns by  $\Delta\theta$  we obtain the condition

$$\frac{J\mu}{L_p}(\psi + \Delta\theta) = r_p(F_n - \frac{\kappa r_p \Delta\theta}{L_n}) \quad (24)$$

with solution

$$\Delta\theta = \frac{r_p F_n - J\mu\psi/L_p}{J\mu/L_p + \kappa r_p^2/L_n}. \quad (25)$$

The new state is now given by  $\hat{F}_s = F_s$ ,  $\hat{\psi} = \psi + \Delta\theta$  and  $\hat{F}_n = F_n - \frac{\kappa r_p \Delta\theta}{L_n}$ .

If the string shifts we first consider the case

$$\hat{F}_n - (1 + \beta)F_s > s_f F_V. \quad (26)$$

The differential tension will now pull the string an amount  $\Delta V$  up over the V-bar until the tension differential reaches  $s_f F_V$ . We now have

$$\frac{J\mu}{L_p}(\psi + \Delta\theta) = r_p(F_n - \frac{\kappa r_p \Delta\theta}{L_n} - \frac{\Delta V}{L_n}). \quad (27a)$$

The condition  $\hat{F}_n - (1 + \beta)F_s = s_f F_V$  translates to

$$F_n - \frac{\kappa r_p \Delta\theta}{L_n} - \frac{\kappa \Delta V}{L_n} - ((1 + \beta)F_s + \frac{\kappa \Delta V}{L_s}) = s_f F_V. \quad (27b)$$

We now have two equations (27) for the two unknowns  $\Delta\theta$  and  $\Delta V$  which we solve. The updated state is now

$$\hat{\psi} = \psi + \Delta\theta \quad (28a)$$

$$\hat{F}_n = F_n - \frac{\kappa r_p \Delta\theta}{L_n} - \frac{\kappa \Delta V}{L_n} \quad (28b)$$

$$\hat{F}_s = F_s + \frac{\kappa \Delta V}{L_s} \quad (28c)$$

If instead  $(1 + \beta)F_s - \hat{F}_n > s_f F_V$  the same formulae apply except for a sign change for  $F_V$  in (27b).

### 6.3 Test blow without releasing the lever

Now we hold the lever in a fixed position and apply a test blow. shift at the V-bar. If it turns out that

$$|F_n - (1 + \beta)F_s| \leq s_f F_V \quad (29)$$

nothing happens. If (29) is not satisfied we have to calculate the shift  $\Delta V$  over the V-bar which will be positive (up) if  $F_n - (1 + \beta)F_s > s_f F_V$  and negative (down) if  $(1 + \beta)F_s - F_n > s_f F_V$ . In that case the new forces are

$$\hat{F}_n = F_n - \kappa \Delta V / L_n. \quad (30)$$

and

$$\hat{F}_s = F_s + \kappa \Delta V / L_s. \quad (31)$$

The tension differential must be maximal (i.e., the shift stops when the static friction stops the process) which gives the condition

$$\hat{F}_n - (1 + \beta)\hat{F}_s = s_f F_V \quad (32)$$

from which we can solve for  $\Delta V$  giving

$$\Delta V = \frac{L_{ns}}{\kappa} (F_n - (1 + \beta)F_s - s_f F_V) \quad (33)$$

New updated forces are now obtained from (30) and (31). Repeating this for the down case ( $(1 + \beta)F_s - F_n > s_f F_V$ ) gives the same formulae except in (33) we have to change the sign in front of  $F_V$ .

## 7 Simulation results

The methods described have been implemented in software using MATLAB. We use the default parameter values for the left string of A4 as indicated in Table 1 unless otherwise indicated. These values were taken from a Heintzmann upright piano. The ‘‘uncertain’’ parameters were obtained by trial and error to obtain the correct pitch changes on test blows with or without releasing the lever when lowering and raising pitch as measured on the Heintzmann upright.

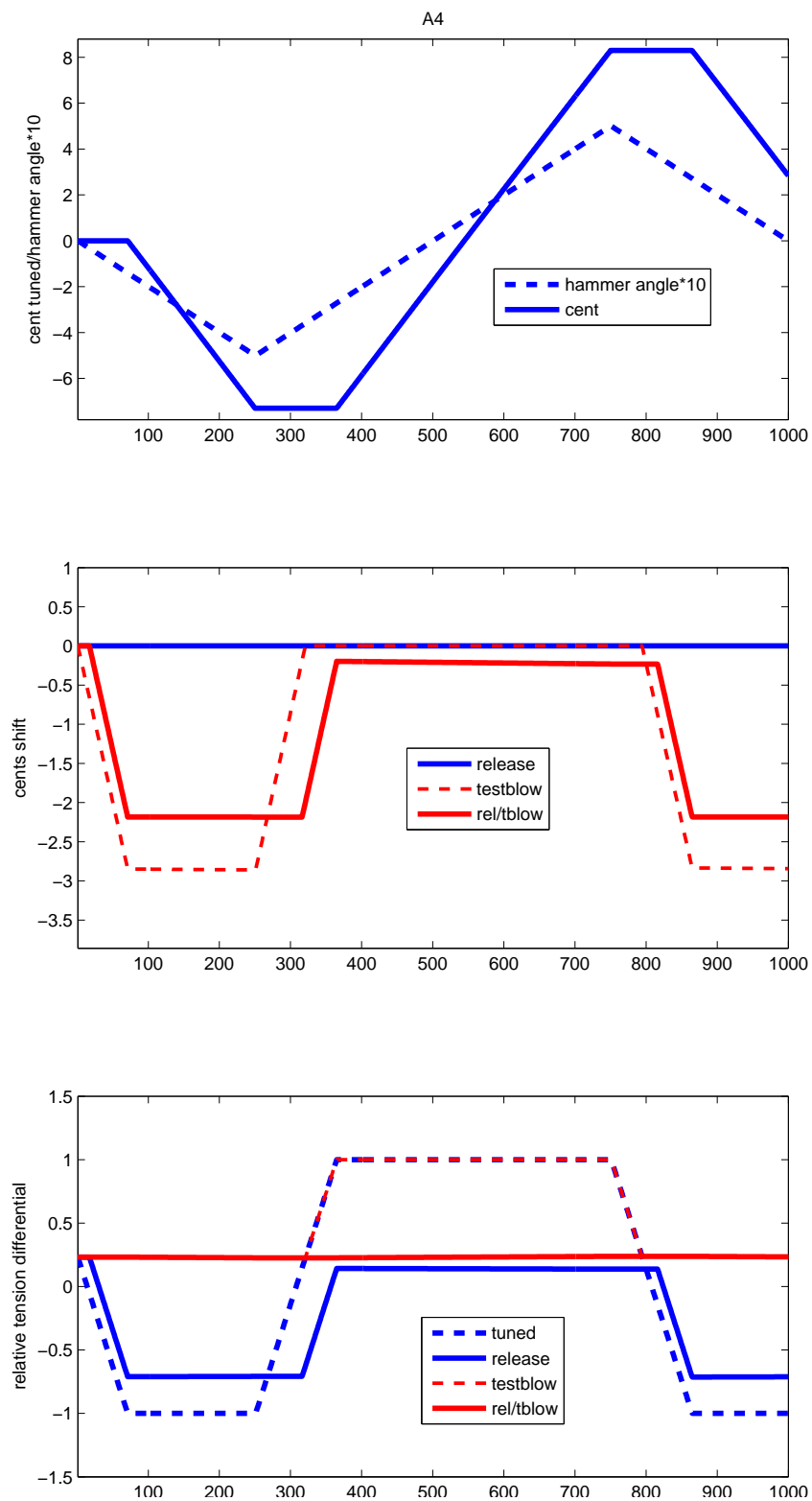


Figure 2: Simulation of tuning A4. Parameters are as in Table 1. Positive angles are clockwise, unlike in the main text.

In Fig. 2 we show the results for a specific lever trajectory. The lever is turned counter clockwise by  $0.5^\circ$ , then  $1^\circ$  in the other direction, and then  $0.5^\circ$  back to the starting point. The upper graph depicts the lever trajectory subdivided in 1000 steps in the dashed line. The vertical scale is multiplied by 10 to allow the simultaneous plot of the cent shift in pitch as heard by softly playing the note, which is the solid line. For instance at step 200 the angle is  $-0.4^\circ$  and the pitch is lowered by  $5.3\text{¢}$ .

The middle graph shows the result of taking one of three actions when the lever is at the corresponding position. The solid blue line indicates the additional pitch change if we just let go of the lever. It is zero everywhere here, but if the pin block was tighter it could be different. The red dashed line shows what would happen if we held the lever fixed at the current position and would apply a test blow. The solid red line shows what would happen if we let go of the lever and then applied a test blow. It is the latter condition that determines the final stable pitch.

Continuing the same example at 200 we see nothing happens if we release the lever, but if a test blow was applied the pitch would drop an additional  $2.9\text{¢}$ , the total pitch change from the beginning thus  $8.2\text{¢}$ . Similarly, releasing the lever followed by a test blow results in an additional pitch drop of  $2.2\text{¢}$ , a total change in pitch of  $7.5\text{¢}$ . This is less of a drop than when the lever was not released, which is caused by the pin untwisting and increasing  $F_n$ , reducing the downward shift across the V-bar upon a test blow.

Finally in the bottom graph we depict the tension differential across the V-bar, relative to its maximum possible value  $F_V$ . The plot labeled “tuned” is the tension when we move the lever in the prescribed trajectory, the other three correspond to the same scenarios as in the middle plot. Note that the solid red line which represents the stable configuration has a non zero tension differential which is caused by the effect of the increased string tension during a test blow. Note that the pitch changes only when the tension differential magnitude is maximal (dotted blue line). When we release the lever when tuning down, the pin twists clockwise and the differential becomes less negative. When tuning up the pin twists much more upon release and the tension differential drops significantly from  $+1$ . The result is a large drop in pitch upon release/test blow when tuning down, but only a negligible drop when tuning up.

If for example the goal was to lower the pitch stably by  $3\text{¢}$  this could be achieved by stopping at time 91, when the pitch has dropped by  $0.8\text{¢}$ . A release/test blow would then drop an additional  $2.2\text{¢}$  to get the desired pitch. Alternatively we could approach the pitch from above by stopping at time 525 when the pitch has dropped  $2.8\text{¢}$  and the release/test blow would cause an additional  $0.2\text{¢}$  drop in pitch. As  $0.2\text{¢}$  is close to the threshold of practical significance a practical method would be to just approach the pitch from below and stop when the desired pitch has been reached. If we decided to approach from above we would have to stop with the note  $2.2\text{¢}$  sharp, which is more difficult.

In practice we would lower the pitch until the note heard is the desired  $3\text{¢}$  flatter

(at time 145) and then turn the lever clockwise to stabilize the pitch. This method is illustrated in Fig.3. Note we have to turn the lever almost back to its original position.

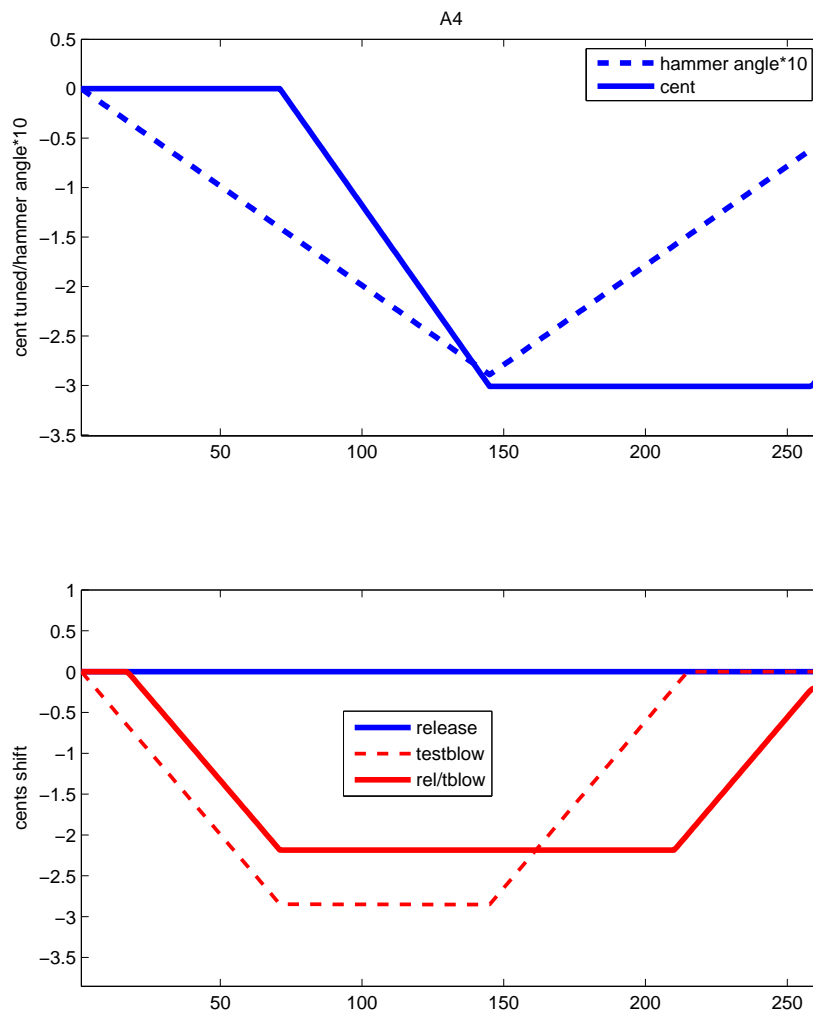


Figure 3: Possible lever motion to stably lower note by  $3\text{c}$ .

## Acknowledgments

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